Low-Complexity Tone Reservation Method for PAPR Reduction of OFDM Systems

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***Abstract*—The OFDM communication system has a serious drawback that the peak signal value can be much higher than the average signal value. High peak signals can be easily distorted by the nonlinearity of power amplifiers, and can increase the symbol error rate (SER) significantly. Though many techniques have been proposed to reduce the peak-to-average-power ratio (PAPR), they are usually based on iterative FFT computation, needing lots of computation time. Based on the tone reservation method, this paper proposes a low complexity DFT structure in order to lower the complexity of PAPR reduction techniques. In the proposed method, several approximations are employed to reduce the computational complexity and to substitute complex multiplications with simple shift operations. The performance of the proposed tone reservation method is compared with that of the conventional method based on the radix-2 FFT. Simulation results show that there is almost no performance degradation if the radix-2 FFT is replaced with the proposed approximate DFT.**

I. INTRODUCTION

The orthogonal frequency division multiplexing (OFDM) suffers from high peak-to-average-power ratio (PAPR) due to large number of sub-carrier signals to be accumulated in time. The OFDM consists of a large number of closely spaced orthogonal sub-carriers to transmit data efficiently. Since the time-domain OFDM signal has a number of subcarriers modulated independently, it can be a high peak value sometimes. Such a high peak signal would be distorted by a non-linear power amplifier. As the number of sub-carriers increases, the probability of high peak signals increases [1]. High PAPR causes three major degradations: increment of symbol error rate (SER), loss of signal power that is called in-band distortion, and increment of interferences among sub-carriers that is called out-of-band distortion.

Many PAPR reduction techniques [2] presented in the literature can be divided into two classes according to the need of side information. The selected mapping (SLM) [2] and the partial transmit sequence (PTS) [2] modify the frequency domain symbols and time-domain OFDM signals, respectively. In this case, the receiver must recover the original signals using the side information to prevent burst errors. As a result, the receiver structure should be modified to make it compatible with the transmitter structure adopting the SLM or PTS technique.

The other class does not need to send side information to a receiver, and includes clipping and filtering (CF) [3], peak windowing (PW) [4], tone reservation (TR) [5-6] and active constellation extension (ACE) [7-8]. The CF and PW reduce peak signals without concerning the change of mapped symbols. Though the out-of-band radiation can be eliminated by employing a filter, the in-band clipping noise that increases SER cannot be eliminated. The TR technique reserves a small number of sub-carriers only for PAPR reduction. This technique does not increase SER, because modulated data symbols are not assigned to the reserved sub-carriers. The ACE method moves symbols adaptively in order not to affect SER by maintaining the distance between any two constellation points. However, this technique requires iterative processing to make the PAPR less than the desired level and increases the average power due to the movement of symbols [3-8]. As a result, the ACE method has a critical drawback of high computational complexity.

Based on the tone reservation method, this paper proposes a low complexity DFT structure in order to lower the

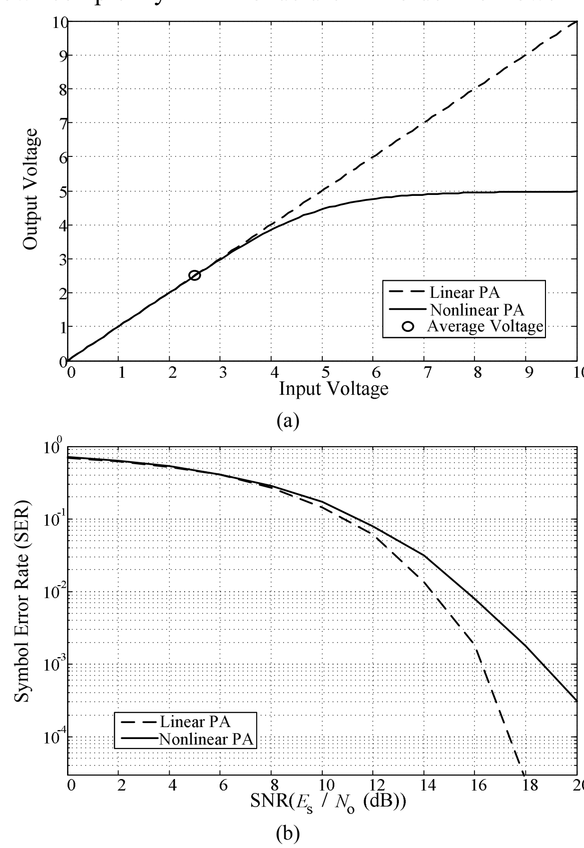

Fig. 1. Non-linear PA characteristics. (a) Saturation of output voltage. (b) SER degradation by the nonlinear distortion.

complexity of PAPR reduction techniques. In the proposed method, several approximations are employed to reduce the computational complexity. The performance of the proposed TR method is comparable with that of the conventional method based on the radix-2 FFT. This paper is organized as follows: Section II defines the PAPR problem and the nonlinear model of power amplifiers. The TR method based on iterative FFT is defined in Section III, and Section IV presents an approximate DFT suitable for the TR method. The simulation results are summarized in Section V and followed by conclusion remarks in Section VI.

II. Definition of PAPR and Nonlinear Power Amplifier Model

The ratio of maximum peak power to the average power in one data block period is referred to as PAPR, and defined as

$$\mathrm{PAPR} = \frac{\max_{0\le t<NT} |x(t)|^2}{1/NT \cdot \int_0^T |x(t)|^2 \, dx} \quad (1)$$

where N is the number of sub-carriers, T is the duration of a data symbol, and $x(t)$ is the complex baseband representation of an OFDM signal. The nonlinearity of a solid state power amplifier (SSPA) [9] is modeled as

$$V_{out} = \frac{V_{in}}{\left(1+\left(|V_{in}|/V_{sat}\right)^{2P}\right)^{1/2P}} \quad (2)$$

where V_{out} and V_{in} are the output and input voltage values, respectively, V_{sat} is the output saturation level. The parameter P is the knee factor that controls the smoothness of the transition from the linear region to the saturation region. One example is shown in Fig. 1(a), where V_{sat} is 6 dB above the average input voltage and P is set to 3. The corresponding SER is presented in Fig. 1(b), where we can see that the SER resulting from the nonlinear power amplifier is larger than that from the linear PA. The SER is closely related to the number of peak signals and the saturation level.

III. Iterative FFT in Tone Reservation

In OFDM systems, a number of sub-carriers (tones) are not allocated to carry data. As these tones are not used for data transmission, they can be mapped to appropriate symbols that can reduce peak signals in the time domain. This principle was introduced in [10] and such sub-carriers are called reserved tones. The tone reservation (TR) method changes the symbols of reserved tones iteratively by using the FFT and the projection onto convex sets (POCS) approach [11]. The POCS-based clipping is widely applied to calculate clipping signals, as it has nice theoretical and optimality properties. When $x_{Clipping}[n]$ represents the clipped-off portion of the signal,

$$x_{Clipping}[n] = \begin{cases} L_{max}\cdot \exp(j\angle x[n]) - x[n], & |x[n]| > L_{max} \\ 0 & |x[n]| \le L_{max} \end{cases} \quad (3)$$

where L_{max} is a threshold level of clipping. Fig. 2 shows the overall block diagram of the TR method associated with iterative FFT.

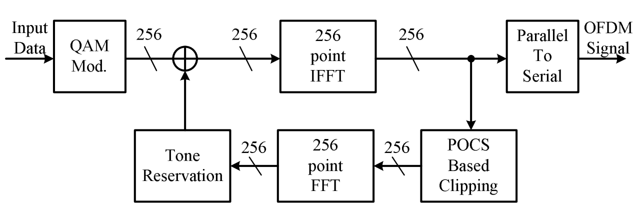

Fig. 2. Tone Reservation technique with iterative FFT.

IV. Approximate DFT

Concerning the implementation of the TR technique for a real-time system, it is time-consuming to calculate FFT exactly. To reduce the high computational complexity of iterative Fourier transform, this paper proposes a DFT that has approximate twiddle factors.

A. The number of inputs and outputs

One important observation in the TR method is that the number of FFT inputs and outputs is much smaller than the number of sub-carriers (N). The DFT defined as

$$X[k] = \sum_{n=0}^{N-1} x[n]\cdot\left(\cos\left(\frac{-2\pi\cdot k\cdot n}{N}\right) + j\sin\left(\frac{-2\pi\cdot k\cdot n}{N}\right)\right), \quad k = 0,1,2,3,\ldots,N-1 \quad (4)$$

requires NxN complex multiplications. However, The number of peak signals exceeding the clipping level is only 32 on the average when N is 256. Therefore, 32x16 complex multiplications are usually sufficient if the number of reserved tones is 16. When N is less than 1024, such a DFT requires a less number of multiplications compared to the radix-2 FFT that is widely used in real implementation.

B. Approximations on the twiddle factor

A complex multiplication can be realized with four real multiplications and two real additions, as expressed in (5). The multiplicand is the time-domain OFDM signal, and the multiplier is a complex exponential called a twiddle factor.

$$\begin{aligned} X[k] &= \sum_n (a_n + jb_n)\cdot\left(\cos\left(\frac{-2\pi\cdot k\cdot n}{N}\right) + j\sin\left(\frac{-2\pi\cdot k\cdot n}{N}\right)\right) \\ &= \sum_n (a_n + jb_n)\cdot W_N^{kn} \end{aligned} \quad (5)$$

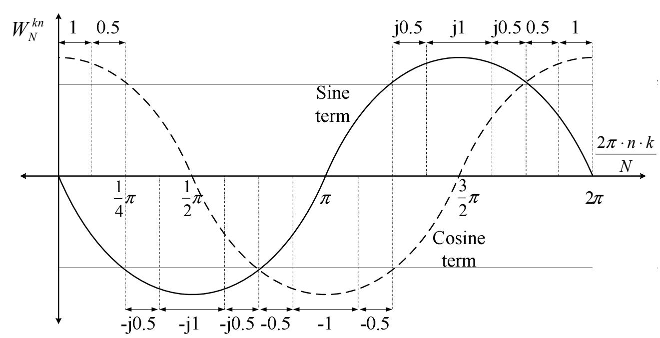

Fig. 3. Approximate twiddle factor.

where k is the index of reserved tones and n is the index of non-zero clipping signals. As the cosine and sine terms of a twiddle factor are orthogonal, one of the two terms plays a major role. Therefore, a twiddle factor can be approximated by taking the major term according to the phase θ of the twiddle factor. In this case, a complex multiplication can be reduced to two real multiplications. In addition to this, no addition is required in the approximate complex multiplication. As a result, the approximate DFT is

$$X[k]=\begin{cases}\sum_n (a_n+jb_n)\cdot\cos\left(\frac{-2\pi\cdot k\cdot n}{N}\right), \\ when\ \theta<\frac{1}{4}\pi, \frac{3}{4}\pi\le\theta<\frac{5}{4}\pi, \theta\ge\frac{7}{4}\pi, \\ \sum_n (a_n+jb_n)\cdot j\cdot\sin\left(\frac{-2\pi\cdot k\cdot n}{N}\right), \\ when \frac{1}{4}\pi\le\theta<\frac{3}{4}\pi, \frac{5}{4}\pi\le\theta<\frac{7}{4}\pi, \theta=\frac{2\pi\cdot k\cdot n}{N}.\end{cases} \quad (6)$$

To reduce the complexity further, we can approximate the magnitude of a twiddle factor to either 1 or 0.5. In this case, the multiplication can be substituted with a shift operation consuming less computation time and less power. Fig. 3 shows how the twiddle factor is approximated according to phase θ, where the whole phase range is divided into 16 regions. For example, θ is in the 3rd region ($\pi/4 \le \theta < 3\pi/8$), the sine term is selected and approximated to -0.5. In that region, the complex multiplication is achieved by shifting multiplicand $x[n]$ right by 1 bit, since the multiplication by 0.5 is equal to shifting right by 1 bit.

C. Lookup table for approximate DFT

After finding non-zero values resulting from POCS-based clipping, a set of approximate twiddle factors is selected from a look-up table (LUT) by using the index of a non-zero value. The set contains all the twiddle factors of k reserved tones. In the conventional DFT algorithm, the operation of nk modulo N is performed to compute the circular index of the LUT. However, k is small enough to store all pre-calculated twiddle factors. Even though the LUT size is proportional to the number of reserved tones, it does not increase much because the approximate twiddle factor can be encoded in 3 bits: two bits to indicate one of {-1, -0.5, +0.5, +1} and an additional bit to indicate which one is dominant between cosine and sine terms. As a result, an entry of the LUT is only 48 bits long, if k is 16. Moreover, it does not require additional calculations for the circular index. Fig. 4 shows such a LUT storing approximate twiddle factors.

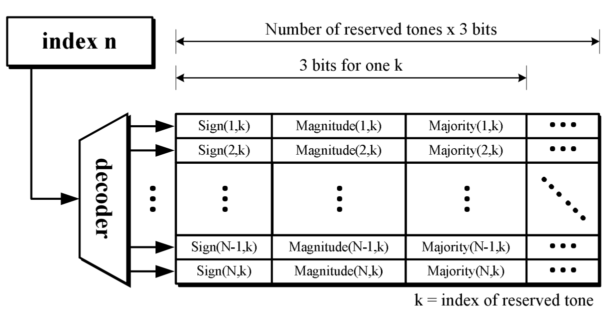

Fig. 4. Look-up table for approximate twiddle factor.

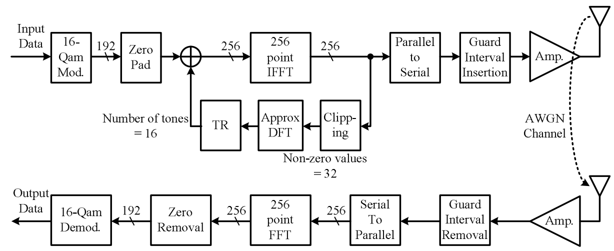

Fig. 5. Tone Reservation technique with approximate DFT.

V. Simulation

The computational complexities of the radix-2 FFT and the proposed DFT are presented in Table I. It is well known that the complexity of the radix-2 FFT is much lower than that of the conventional DFT, especially for large inputs. However, the proposed DFT not only has a less number of inputs and outputs but also replaces multiplications with shift operations. Table II summarizes the number of computations required for several values of N. Table II also shows how the number of non-zero inputs is related to the data block size (N) if the clipping level (L_{max}) is set to 3 dB above the average signal power. From the relationship, we assume that number of reserved tones is proportional to N and is about a half of the number of non-zero inputs.

To evaluate the performance of the proposed DFT, simulation has been conducted for a communication system modeled as follows. There are 256 symbols in a data block: 192 symbols are 16-QAM modulated data and 64 symbols are filled with zeros. The time-domain OFDM signal is computed by 256-point IFFT. After several iterations, the modified OFDM signal is serialized and followed by the guard interval (GI) insertion. Then it is transmitted through an AWGN channel. In the receiver, the received signal is converted into symbols by passing through the GI removal block and FFT unit. The received data symbols are then demodulated by

TABLE I. Computational Complexity of Radix-2 FFT and Proposed DFT

Method		Computational Complexity
Radix-2 FFT	Complex Multiplication	$N/2 \times \log_2 N$
	Complex Addition	$N \times \log_2 N$
Proposed DFT	Shift	(Number of non-zero input) x(Number of reserved tones)
	Real Addition	(Number of non-zero input) x(Number of reserved tones)

TABLE II. Number of Computations for Radix-2 FFT and Proposed DFT

N		64	128	256	512	1024
Number of non-zero input		8	16	32	60	122
Number of reserved tones		4	8	16	32	64
Radix-2 FFT	Complex Multiplication	192	448	1024	2304	5120
	Complex Addition	384	896	2048	4608	10240
Proposed DFT	Shift	32	128	512	1920	7808
	Real Addition	32	128	512	1920	7808

TABLE III. AVERAGE AND PEAK POWER OF RADIX-2 FFT AND PROPOSED DFT

Method	Average Power	Peak Power
Original	7.48	45.17
Radix-2 FFT with 8 tones	7.50 (0.3%)	42.76 (-5.3%)
Pseudo DFT with 8 tones	7.63 (2.0%)	40.99 (-9.3%)
Radix-2 FFT with 16 tones	7.54 (0.8%)	39.71 (-12.1%)
Pseudo DFT with 16 tones	7.78 (4.0%)	36.83 (-18.4%)

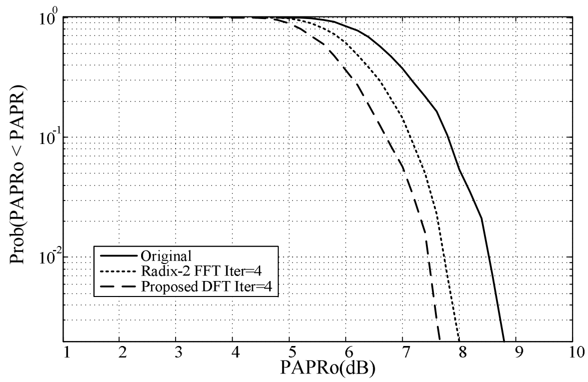

Fig. 6. PAPR reduction comparison of radix-2 FFT and proposed DFT with 4 iterations.

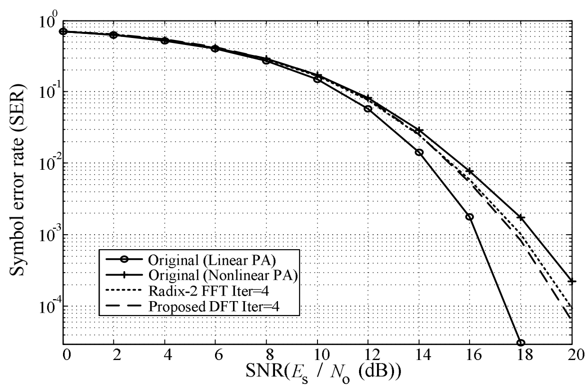

Fig. 7. SER comparison of radix-2 FFT and proposed DFT with 4 iterations.

applying the hard decision of 16-QAM.

Fig. 6 shows the cumulative distribution function (CDF) resulting from the radix-2 FFT and the proposed DFT, where the horizontal axis is the threshold PAPR value and the vertical axis represents the probability that the PAPR of an OFDM signal exceeds a certain PAPR value, that is,

$$\mathrm{CDF(PAPR)} = prob(\mathrm{PAPR} > \mathrm{PAPR_O}) \,. \qquad (7)$$

Table III shows the average power and the peak power achieved with no PAPR reduction method (denoted as original in the table) and with the TR method. The TR results are obtained by using the conventional FFT and proposed DFT. Compared to the radix-2 FFT, the proposed DFT slightly increases the average power by 3.2% and decreases the peak power by 6.3% when 16 tones are reserved for PAPR reduction. For fair comparison, a scaling factor is introduced to reduce the displacement of frequency domain symbols per iteration, which makes the peak signal reduction similar to that of the radix-2 FFT. The average power with the scaling factor is 7.57, which is very similar to those achieved by the original and the radix-2 FFT. As a result, the approximate DFT does not impair the PAPR reduction capability and the average power. Fig. 7 shows SER graphs resulting from the TR methods based on the conventional FFT and the proposed DFT, where we can see SER results are similar to each other. Reducing high peak signal makes the graph move towards the SER achieved with the linear PA.

VI. CONCLUSION

The OFDM system suffers from high peak-to-average-power ratio (PAPR) that produces distortion in the nonlinear power amplifier. The proposed PAPR reduction technique is grounded on the tone reservation method. In the tone reservation method, the computational complexity is significantly reduced by approximating twiddle factors needed in the DFT. More specifically, it reduces the number of calculations and changes multiplications to shift operations while maintaining the performance of SER and PAPR reduction at the same level of the radix-2 FFT. The proposed DFT can be applied to other PAPR techniques requiring iterative FFT such as active constellation extension, recursive clipping and filtering, and recursive peak windowing.

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